Simulating Quantum Noise and Anti-Squeezing in Kerr Media Using Stochastic Methods

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Guiding Question

Can we create a simulation that finds the most optimal experimental conditions to experimentally invoke Kerr Squeezing to develop new quantum spectroscopy techniques?

Understanding Quantum Noise & Kerr Squeezing

Kerr squeezing arises from the nonlinear interaction of light in optical fibers. When intense pulses propagate through a medium with a third-order nonlinearity ($\chi^{(3)}$), phase-dependent quantum noise can be reduced below the standard quantum limit. This effect enables the generation of squeezed states of light, which are applied in cutting-edge experiments such as LIGO, where squeezed light enhances gravitational wave measurements by reducing quantum noise below the standard quantum limit.

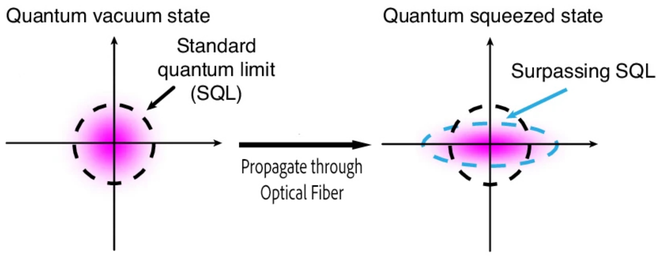


Figure 1. A conceptual visualization of quantum noise squeezing, where a vacuum state undergoes nonlinear propagation through an optical fiber, resulting in reduced noise along one quadrature - represented by a narrowed uncertainty ellipse that surpasses the standard quantum limit (SQL).[1]

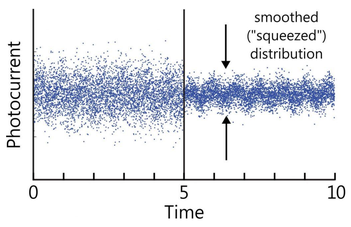


Figure 2. An experimental-like representation of quantum noise squeezing in the time domain: the photocurrent noise distribution narrows after propagation, indicating reduced quantum uncertainty in the measured quadrature.[2]

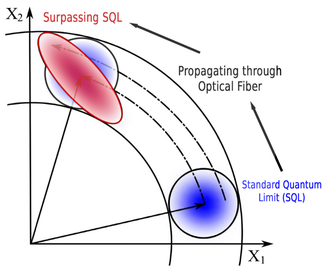


Figure 3. A conceptual diagram of Kerr squeezing, where the noise ellipse rotates in phase space due to nonlinear phase shifts during propagation through a $\chi^{(3)}$ medium. This rotation reflects the evolving squeezing angle, with clear compression along one quadrature and expansion (anti-squeezing) along the other.[3]

PyNLO Package Modifications

PyNLO[7] is a Python package for modeling nonlinear light–matter interactions. We used it to simulate pulse propagation through a $\chi^{(3)}$ medium. The original code solves the classical NLSE using the split-step Fourier method (SSFM). To model Kerr squeezing, we modified it to solve the quantum stochastic NLSE.

NLSE (classical)

$$\frac{\partial}{\partial z}A(t,z) = i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A(t,z) + \left[i\gamma\int_0^\infty dt' R(t-t')\left|A(t',z)\right|^2\right]A(t,z) - \frac{\alpha}{2}A(t,z)$$

Dispersion (linear) — Kerr & Raman (nonlinear) — Absorption (linear)

Stochastic NLSE (quantum)

$$\frac{\partial}{\partial z}A(t,z) = i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A(t,z) + \left[i\gamma\int_0^\infty dt' R(t-t')\left|A(t',z)\right|^2 + i\Gamma^R(t,z)\right]A(t,z) - \frac{\alpha}{2}A(t,z) + \Gamma(t,z),$$

Nonlinear noise — Linear noise

Figure 4. In the SSFM, we extended the classical NLSE to include both linear and nonlinear noise terms, enabling simulation of the quantum stochastic NLSE. [6]

Simulation Development & Analysis

Simulation Parameters: We simulated 200 fs pulses at 1550 nm propagating through a 50 m optical fiber using the modified SSFM. Dispersion parameters were set to $\beta_2 = -10.5\,\mathrm{ps}^2/\mathrm{km}$ and $\beta_3 = 0.155\,\mathrm{ps}^3/\mathrm{km}$, with Kerr nonlinearity $\Gamma = 3\,\mathrm{W}^{-1}\mathrm{km}^{-1}$. Raman scattering and self-steepening were enabled, and results were averaged over 500 noise realizations.

Noise Plotting: The simulation was performed using the specified parameters, with each realization capturing the input and output quantum noise in phase space, shown below. Both graph has had their data points centered at (0,0), which makes the squeezing angle less obvious. Through comparison, the graphs reveals that the semi-major axis (red arrow) is stretched to by approximately 0.04 a.u., indicating significant anti-squeezing along this quadrature.

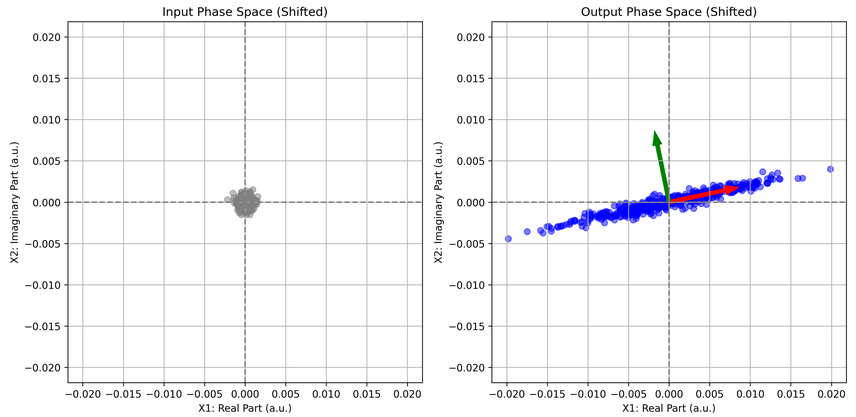


Figure 5. Input & Output Graphs of Quantum Noise of Pulses propagating through an Optical Fiber
The green arrow represents the squeezing level (semi-minor axis) and the red arrow represents the anti-squeezing level (semi-major axis).

Visualizing through Color Map: After developing a reliable simulator, we utilized it to create a 3D dataset, with squeezing calculated as a function of pulse duration, pulse energy, and fiber length. The simulation was carried out on a 16×16 grid — each point recording 50 realizations — but we have used data interpolation for a better visual representation. From observation, the anti-squeezing levels shift to the left side of the color map as we increase the fiber length.

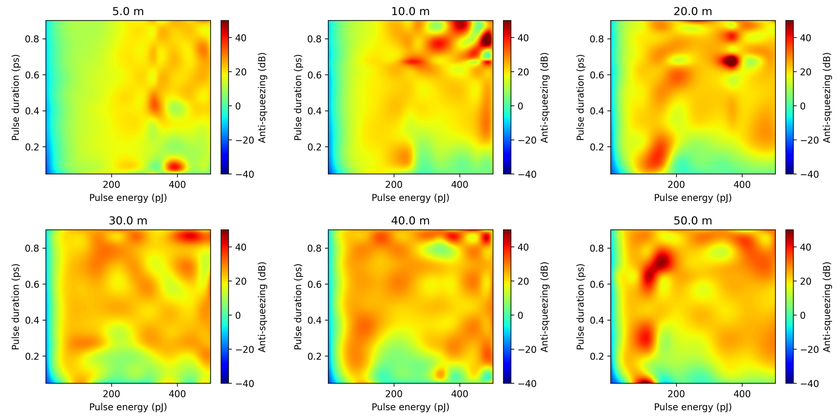
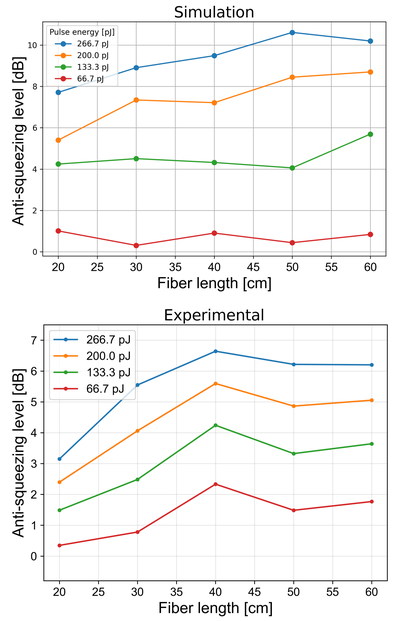


Figure 6. Color Maps for different fiber lengths from 5.0 to 50.0 meters simulating squeezing for different pulse energies and durations. [4,5]

Experimental Results

We collected experimental data on anti-squeezing as a function of fiber length for four pulse energies and compared the results with our simulation (top: simulation; bottom: experiment). In both experiment and simulation, higher pulse energy and longer fiber length yield higher anti-squeezing. The experimental curves track the simulated ordering and length dependence, demonstrating qualitative consistency between model and measurement. Minor offsets are expected from propagation loss and phase-noise effects not fully captured in the model.



- **Pulse duration (FWHM):** 20 fs
- **Central wavelength:** 800 nm
- **Central frequency:** ~ 374.7 THz
- **Attenuation coefficient:** $\alpha = 0.23\,\mathrm{m}^{-1}$ (equivalent to $1\,\mathrm{dB/km} \approx 0.023\%$ power loss per meter)
- **Group velocity dispersion:** $\beta_2 = 38.1\,\mathrm{ps}^2/\mathrm{km}$
- **Third-order dispersion:** $\beta_3 = 0.1\,\mathrm{ps}^3/\mathrm{km}$
- **Nonlinear refractive index:** $n_2 = 2.7 \times 10^{-20}\,\mathrm{m}^2/\mathrm{W}$
- **Mode field diameter:** $5\,\mu\mathrm{m}$
- **Nonlinear coefficient:** $\Gamma \approx 8.51\,\mathrm{W}^{-1}\mathrm{km}^{-1}$
- **Simulation window:** 1.0 ps with 2^{10} points
- **Fiber length:** 0.4 m (can be varied)
- **Steps along fiber:** 100 (may auto-adjust in SSFM)
- **Raman effect:** Included
- **Self-steepening:** Included
- **Realizations for noise:** 100

Figure 7. Experimental vs. Simulated Data graphed for Anti-Squeezing vs. Fiber Length on the left. Parameters for the experimental environment used are listed on the right.

Conclusion

We successfully developed a Kerr squeezing simulator that helps find out optimal conditions for Kerr squeezing experiments. It visualizes input and output noise distributions, along with qualitative analysis on anti-squeezing level when sweeping parameter over pulse energy, duration, and propagation length. The developed simulator successfully predicts the tendency of anti-squeezing level when increasing pulse energy at different fiber lengths and matches with Kerr squeezing experiments, validating the simulator's performance. Further work is needed to accurately model squeezing levels and refine parameters for maximizing noise squeezing.

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